Math 42 Midterm 3 Review

You will need a calculator to solve the problems marked ★.
You should **NOT** use a calculator for any other problems.

[0] Write out all the basic identities:

[a] $\sin(-x) =$ $\cos(-x) =$ $\tan(-x) =$

$\sec(-x) =$ $\csc(-x) =$ $\cot(-x) =$

[b] $\sin x = \frac{1}{\quad}$ $\cos x = \frac{1}{\quad}$ $\tan x = \frac{1}{\quad}$

$\sec x = \frac{1}{\quad}$ $\csc x = \frac{1}{\quad}$ $\cot x = \frac{1}{\quad}$

[c] $\tan x = \frac{x}{x}$ $\cot x = \frac{x}{x}$

[d] $^{2}x + \ ^{2}x = 1$ $^{2}x - \ ^{2}x = 1$ **2 versions**

$1 + \ ^{2}x = \ ^{2}x$ **2 versions** $1 - \ ^{2}x = \ ^{2}x$ **2 versions** $^{2}x - 1 = \ ^{2}x$ **2 versions**

[e] $\sin(x+y) =$ $\cos(x+y) =$ $\tan(x+y) =$

$\sin(x-y) =$ $\cos(x-y) =$ $\tan(x-y) =$

[f] $\sin 2x =$ $\cos 2x =$ **3 versions** $\tan 2x =$

[g] $\sin^2 x =$ $\cos^2 x =$ **These answers should not contain the square of any function**

[h] $\sin \frac{1}{2}x =$ $\cos \frac{1}{2}x =$ $\tan \frac{1}{2}x =$ **3 versions**

[00] Prove the most difficult looking identities in the textbook exercises from sections 5.2, 5.4, 5.5, chapter 5 review, chapter 5 test and additional homework assignment #2.

[1] Simplify the following expressions completely.
A product of two factors is simpler than a sum of two terms.
There should be as few terms, factors and trigonometric functions as possible.
Exponents should be as small as possible (without using negative exponents).

[a] $\sec x \cot x \sin x$ [b] $(1-\sec x)(1+\cos x)$ [c] $\dfrac{\sec x}{\csc x}+\dfrac{\cos x}{\sin x}$

[d] $2\cot 2x-\cot x+\tan x$ [e] $6\cos\frac{7\pi}{12}\sin\frac{7\pi}{12}$ [f] $5-10\cos^2\frac{3\pi}{8}$

[g] $\sin^2 165^\circ$ [h] $\dfrac{\tan^2 75^\circ-1}{\tan 75^\circ}$ [i] $\cos 105^\circ \sin 15^\circ+\sin 105^\circ \cos 15^\circ$

[j] $\sin\frac{\pi}{4}\sin\frac{11\pi}{12}+\cos\frac{\pi}{4}\cos\frac{11\pi}{12}$

[2] If $\sec x=-\frac{7}{5}$ and $\pi<x<\frac{3\pi}{2}$, and $\cot y=-\frac{3}{4}$ and $\sin y>0$, find the values of the following expressions.
Simplify your final answers.

[a] $\sin 2y$ [b] $\tan(x-y)$ [c] $\cos\frac{x}{2}$

[d] $\cos(x-\frac{5\pi}{3})$ [e] $\csc\frac{x}{2}$ [f] $\tan 2x$

[g] $\sin(y+\arccos(-\frac{1}{5}))$ [h] $\sec(x+y)$ [i] $\cos 2y$

[3] Use the power reducing formulae to rewrite the following expressions using only the first powers of cosine.
Your final answers must **NOT** be in factored form. Simplify your final answers.

[a] $\sin^4 x$ [b] $\sin^2 x\cos^4 x$

[4] Rewrite the following expressions using only $\sin x$, $\cos x$ and/or $\tan x$.
Simplify your final answers to use as few trigonometric functions as possible.

[a] $\sin 3x$ [b] $\tan 4x$ [c] $\cos 4x$

[5] Find the value of the following expressions. Simplify your final answers.

[a] $\cos(\arctan(-2)-\sin^{-1}(-\frac{3}{7}))$ [b] $\sin(2\tan^{-1}3)$ [c] $\tan(\sin^{-1}\frac{1}{3}-\arccos(-\frac{2}{3}))$

[6] Solve the following equations.

[a] $\tan x+3\sqrt{3}=2(\sqrt{3}-\tan x)$ [b] $5(1+\csc x)=4-3\cot^2 x$ [c] $4\cos 4x+2\sqrt{2}=0$

[d] $\sqrt{3}\cot\frac{1}{3}x-3=0$ [e] $\sin 2x+\sqrt{3}\cos x=0$ [f] $3\cos\frac{1}{2}x-2\cos x=1$ ★